3.2.1 EXERCISES

For a link to all of the additional resources available for this section, click OSttS Chapter 3 materials.

In Exercises 1 - 6, use polynomial long division to perform the indicated division. Write the polynomial in the form $p(x) = d(x)q(x) + r(x)$.

For help with these exercises, click on the resource below:

- How to perform polynomial long division

1. $\left(4x^2 + 3x - 1\right) \div (x - 3)$
2. $\left(2x^3 - x + 1\right) \div \left(x^2 + x + 1\right)$
3. $\left(5x^4 - 3x^3 + 2x^2 - 1\right) \div \left(x^2 + 4\right)$
4. $\left(-x^5 + 7x^3 - x\right) \div \left(x^3 - x^2 + 1\right)$
5. $\left(9x^3 + 5\right) \div (2x - 3)$
6. $\left(4x^2 - x - 23\right) \div \left(x^2 - 1\right)$

In Exercises 7 - 20 use synthetic division to perform the indicated division. Write the polynomial in the form $p(x) = d(x)q(x) + r(x)$.

For help with these exercises, click on the resource below:

- How to perform synthetic division

7. $\left(3x^2 - 2x + 1\right) \div (x - 1)$
8. $\left(x^2 - 5\right) \div (x - 5)$
9. $\left(3 - 4x - 2x^2\right) \div (x + 1)$
10. $\left(4x^2 - 5x + 3\right) \div (x + 3)$
11. $\left(x^3 + 8\right) \div (x + 2)$
12. $\left(4x^3 + 2x - 3\right) \div (x - 3)$
13. $\left(18x^2 - 15x - 25\right) \div \left(x - \frac{5}{3}\right)$
14. $\left(4x^2 - 1\right) \div \left(x - \frac{1}{2}\right)$
15. $\left(2x^3 + x^2 + 2x + 1\right) \div \left(x + \frac{1}{2}\right)$
16. $\left(3x^3 - x + 4\right) \div \left(x - \frac{2}{3}\right)$
17. $\left(2x^3 - 3x + 1\right) \div \left(x - \frac{1}{2}\right)$
18. $\left(4x^4 - 12x^3 + 13x^2 - 12x + 9\right) \div \left(x - \frac{3}{2}\right)$
19. $\left(x^4 - 6x^2 + 9\right) \div \left(x - \sqrt{3}\right)$
20. $\left(x^6 - 6x^4 + 12x^2 - 8\right) \div \left(x + \sqrt{2}\right)$

In Exercises 21 - 30, determine $p(c)$ using the Remainder Theorem for the given polynomial functions and value of c. If $p(c) = 0$, factor $p(x) = (x - c)q(x)$.

For help with these exercises, click on the resource below:

- Using the Remainder Theorem

21. $p(x) = 2x^2 - x + 1$, $c = 4$
22. $p(x) = 4x^2 - 33x - 180$, $c = 12$
23. $p(x) = 2x^3 - x + 6$, $c = -3$
24. $p(x) = x^3 + 2x^2 + 3x + 4$, $c = -1$
25. $p(x) = 3x^3 - 6x^2 + 4x - 8$, $c = 2$
26. $p(x) = 8x^3 + 12x^2 + 6x + 1$, $c = -\frac{1}{2}$
27. $p(x) = x^4 - 2x^2 + 4$, $c = \frac{3}{2}$
28. $p(x) = 6x^4 - x^2 + 2$, $c = -\frac{2}{3}$
29. $p(x) = x^4 + x^3 - 6x^2 - 7x - 7$, $c = -\sqrt{7}$
30. $p(x) = x^2 - 4x + 1$, $c = 2 - \sqrt{3}$

In Exercises 31 - 40, you are given a polynomial and one of its zeros. Use the techniques in this section to find the rest of the real zeros and factor the polynomial.

For help with these exercises, click on the resource below:

- Using the Factor Theorem

31. $x^3 - 6x^2 + 11x - 6$, $c = 1$
32. $x^3 - 24x^2 + 192x - 512$, $c = 8$
33. $3x^3 + 4x^2 - x - 2$, $c = \frac{2}{3}$
34. $2x^3 - 3x^2 - 11x + 6$, $c = \frac{1}{2}$
35. $x^3 + 2x^2 - 3x - 6$, $c = -2$
36. $2x^3 - x^2 - 10x + 5$, $c = \frac{1}{2}$
37. $4x^4 - 28x^3 + 61x^2 - 42x + 9$, $c = \frac{1}{2}$ is a zero of multiplicity 2
38. $x^5 + 2x^4 - 12x^3 - 38x^2 - 37x - 12$, $c = -1$ is a zero of multiplicity 3
39. $125x^5 - 275x^4 - 2265x^3 - 3213x^2 - 1728x - 324$, $c = -\frac{3}{5}$ is a zero of multiplicity 3
40. $x^2 - 2x - 2$, $c = 1 - \sqrt{3}$

In Exercises 41 - 45, create a polynomial p which has the desired characteristics. You may leave the polynomial in factored form.

41.
- The zeros of p are $c = \pm 2$ and $c = \pm 1$
- The leading term of $p(x)$ is $117x^4$.

42.
- The zeros of p are $c = 1$ and $c = 3$
- $c = 3$ is a zero of multiplicity 2.
- The leading term of $p(x)$ is $-5x^3$

43.
- The solutions to $p(x) = 0$ are $x = \pm 3$ and $x = 6$
- The leading term of $p(x)$ is $7x^4$
- The point $(-3, 0)$ is a local minimum on the graph of $y = p(x)$.

44.
- The solutions to $p(x) = 0$ are $x = \pm 3$, $x = -2$, and $x = 4$.
- The leading term of $p(x)$ is $-x^5$.
- The point $(-2, 0)$ is a local maximum on the graph of $y = p(x)$.

45.
- p is degree 4.
- as $x \to \infty$, $p(x) \to -\infty$
- p has exactly three x-intercepts: $(-6, 0)$, $(1, 0)$ and $(117, 0)$
- The graph of $y = p(x)$ crosses through the x-axis at $(1, 0)$.

46. Find a quadratic polynomial with integer coefficients which has $x = \frac{3}{5} \pm \frac{\sqrt{29}}{5}$ as its real zeros.

Checkpoint Quiz 3.2

1. Let $p(x) = 3x^3 + x^2 - 21x - 7$.
   (a) Find $p(2)$ using the Remainder Theorem.
   (b) Show $c = -\frac{1}{3}$ is a zero of p and use this to find the remaining zeros of p.

2. Find the zeros of $p(x) = x^2 - 2x - 4$ and use them to factor $p(x)$.

For worked out solutions to this quiz, click the link below:

- Quiz Solution

3.2.2 Answers

1. $4x^2+3x-1=(x-3)(4x+15)+44$
2. $2x^3-x+1=\left(x^2+x+1\right)(2x-2)+(-x+3)$
3. $5x^4-3x^3+2x^2-1=\left(x^2+4\right)\left(5x^2-3x-18\right)+(12x+71)$
4. $-x^5+7x^3-x=\left(x^3-x^2+1\right)\left(-x^2-x+6\right)+\left(7x^2-6\right)$
5. $9x^3+5=(2x-3)\left(\frac{9}{2}x^2+\frac{27}{4}x+\frac{81}{8}\right)+\frac{283}{8}$
6. $4x^2-x-23=\left(x^2-1\right)(4)+(-x-19)$
7. $\left(3x^2-2x+1\right)=(x-1)(3x+1)+2$
8. $\left(x^2-5\right)=(x-5)(x+5)+20$
9. $\left(3-4x-2x^2\right)=(x+1)(-2x-2)+5$
10. $\left(4x^2-5x+3\right)=(x+3)(4x-17)+54$
11. $\left(x^3+8\right)=(x+2)\left(x^2-2x+4\right)+0$
12. $\left(4x^3+2x-3\right)=(x-3)\left(4x^2+12x+38\right)+111$
13. $\left(18x^2-15x-25\right)=\left(x-\frac{5}{3}\right)(18x+15)+0$
14. $\left(4x^2-1\right)=\left(x-\frac{1}{2}\right)(4x+2)+0$
15. $\left(2x^3+x^2+2x+1\right)=\left(x+\frac{1}{2}\right)\left(2x^2+2\right)+0$
16. $\left(3x^3-x+4\right)=\left(x-\frac{2}{3}\right)\left(3x^2+2x+\frac{1}{3}\right)+\frac{38}{9}$
17. $\left(2x^3-3x+1\right)=\left(x-\frac{1}{2}\right)\left(2x^2+x-\frac{5}{2}\right)-\frac{1}{4}$
18. $\left(4x^4-12x^3+13x^2-12x+9\right)=\left(x-\frac{3}{2}\right)\left(4x^3-6x^2+4x-6\right)+0$
19. $\left(x^4-6x^2+9\right)=\left(x-\sqrt{3}\right)\left(x^3+\sqrt{3}\,x^2-3x-3\sqrt{3}\right)+0$
20. $\left(x^6-6x^4+12x^2-8\right)=\left(x+\sqrt{2}\right)\left(x^5-\sqrt{2}\,x^4-4x^3+4\sqrt{2}\,x^2+4x-4\sqrt{2}\right)+0$
21. $p(4)=29$
22. $p(12)=0$, $p(x)=(x-12)(4x+15)$
23. $p(-3)=-45$
24. $p(-1)=2$
25. $p(2)=0$, $p(x)=(x-2)\left(3x^2+4\right)$
26. $p\left(-\frac{1}{2}\right)=0$, $p(x)=\left(x+\frac{1}{2}\right)\left(8x^2+8x+2\right)$

27. $p\left(\frac{3}{2}\right) = \frac{73}{16}$

28. $p\left(-\frac{2}{3}\right) = \frac{74}{27}$

29. $p(-\sqrt{7}) = 0$, $p(x) = (x+\sqrt{7})\left(x^3 + (1-\sqrt{7})x^2 + (1-\sqrt{7})x - \sqrt{7}\right)$

30. $p(2-\sqrt{3}) = 0$, $p(x) = (x-(2-\sqrt{3}))(x-(2+\sqrt{3}))$

31. $x^3 - 6x^2 + 11x - 6 = (x-1)(x-2)(x-3)$

32. $x^3 - 24x^2 + 192x - 512 = (x-8)^3$

33. $3x^3 + 4x^2 - x - 2 = 3\left(x - \frac{2}{3}\right)(x+1)^2$

34. $2x^3 - 3x^2 - 11x + 6 = 2\left(x - \frac{1}{2}\right)(x+2)(x-3)$

35. $x^3 + 2x^2 - 3x - 6 = (x+2)(x+\sqrt{3})(x-\sqrt{3})$

36. $2x^3 - x^2 - 10x + 5 = 2\left(x - \frac{1}{2}\right)(x+\sqrt{5})(x-\sqrt{5})$

37. $4x^4 - 28x^3 + 61x^2 - 42x + 9 = 4\left(x - \frac{1}{2}\right)^2(x-3)^2$

38. $x^5 + 2x^4 - 12x^3 - 38x^2 - 37x - 12 = (x+1)^3(x+3)(x-4)$

39. $125x^5 - 275x^4 - 2265x^3 - 3213x^2 - 1728x - 324 = 125\left(x + \frac{3}{5}\right)^3(x+2)(x-6)$

40. $x^2 - 2x - 2 = (x-(1-\sqrt{3}))(x-(1+\sqrt{3}))$

41. $p(x) = 117(x+2)(x-2)(x+1)(x-1)$

42. $p(x) = -5(x-1)(x-3)^2$

43. $p(x) = 7(x+3)^2(x-3)(x-6)$

44. $p(x) = -(x+2)^2(x-3)(x+3)(x-4)$

45. $p(x) = a(x+6)^2(x-1)(x-117)$ or $p(x) = a(x+6)(x-1)(x-117)^2$ where a is any real number as long as $a < 0$

46. $p(x) = 5x^2 - 6x - 4$